

# Asymptotic Analysis of Three-Dimensional Pressure Interference Tests: A Point-Source Solution

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## Abstract

Well test analyses can be conducted in either steady-state or transient flow regimes. In the former, the pressure transient data collected during a pressure interference test must reach a steady-state. Likewise, in transient type-curve and numerical inverse approaches, the time-drawdown data must fit the model developed for the situation under consideration for the parameter estimates to be meaningful. In many cases these requirements are difficult to meet under field conditions due to external forcings and heterogeneities of the rock properties. Here, we develop a new approach to estimate permeability and porosity from well tests using the asymptotic straight line analysis of pressure transients during three-dimensional pressure interference tests. We apply our newly developed technique to several cross-hole pneumatic injection tests conducted by Illman et al., [1998; see also Illman, 1999] at the Apache Leap Research Site near Superior, Arizona, USA to obtain permeabilities and porosities from these tests. We then compare these results to previously obtained estimates of permeabilities and porosities from type-curve [Illman and Neuman, 2001] and numerical inverse [Vesselinov et al. 2001a,b] analyses and permeabilities from steady-state [Illman and Neuman, 2003] analysis. The comparisons reveal that the newly developed approach yields reliable estimates of permeabilities and porosities from three-dimensional pressure interference tests.

## 1 Introduction

Traditional methods of well test analysis rely on steady-state or transient methods. For the steady-state approach, the pressure transient data collected during a pressure interference

test must reach a steady-state for the method to be applicable. These conditions in many cases are difficult to achieve because the pressure interference tests may have to be run for an exceedingly long time for steady-state conditions to develop. Even after running such a test for a long time, the pressure transients may never reach a steady state. In fact, well tests seldom reach a steady-state making the application of steady-state methods problematic. In addition, the steady-state analysis of pressure interference tests yields only the permeability but not estimates of porosity because of the reliance on the steady state portion of the data for the analysis. These are some important reasons why transient methods such as type-curve analysis, semi-log analysis (Cooper and Jacob, 1946), and numerical inverse modeling approaches have been developed to analyze the transient portion of the data.

Various type-curve models developed for different hydrogeologic conditions allowed for the transient analysis of the time-drawdown data. For the technique to be applicable and the parameter estimates derived from the technique to be meaningful, the time-drawdown data must fit the type-curves developed for the situation under consideration. In many cases these requirements are difficult to meet under field conditions due to factors that complicate the analysis. External factors such as recharge and barometric pressure fluctuations can corrupt the pressure transients making well test interpretation by means of traditional techniques difficult. Likewise, pressure transient data obtained from well tests conducted in heterogeneous media frequently do not match type-curves developed under the assumption that the medium is homogeneous. These complications limit the use of analytical type-curve approaches to simple situations. Numerical inverse approaches can overcome many of these difficulties by incorporating the effects of external forcings and heterogeneities, among other things, but these models can be complex and time-consuming to develop. Therefore, there is a need for alternative yet complementary interpretive approaches for the analysis of pressure interference tests to yield reliable estimates of flow parameters.

The objectives of this paper are to present a new approach to estimate permeability and porosity from three-dimensional pressure interference tests by analyzing the intermediate to late data through a simple graphical technique based on their asymptotic analysis. We apply the technique to previously conducted cross-hole pneumatic injection tests by Illman et al., [1998; see also Illman, 1999] and compare these results to previously obtained estimates of permeabilities and porosities from type-curve [Illman and Neuman, 2001] and numerical inverse [Vesselinov et al. 2001a,b] analyses and permeabilities from steady-state [Illman and Neuman, 2003] analysis.

## 2 Methodology

The methodology rests on obtaining a large time approximation to the point source solution. Here, we develop such an approximation to analyze three-dimensional pressure transient tests conducted using air as a flowing fluid in unsaturated geologic media. We note, however, that the approximation is valid for the interpretation of pressure interference tests in saturated media as well.

The equations that describe airflow in partially saturated porous media are nonlinear due to the compressible nature of air, its capillary interaction with water, and non-Darcian

behavior at high Reynolds numbers. A complete description of air-water interaction requires two systems of coupled partial differential equations, one for each phase. The development of corresponding analytical formulae requires that two-phase flow is approximated as single-phase airflow and that water is treated as immobile. The airflow equation must additionally be linearized to allow solving it either in terms of pressure,  $p$ , as is customary for liquids or in terms of pressure-squared,  $p^2$ , as is more common for gases. Details to the theoretical development are provided in Illman and Neuman [2001]. Illman and Neuman [2000] have shown that interpreting single-hole pneumatic injection tests at the ALRS by means of  $p^2$ -based and  $p$ -based type curves leads to similar results. Illman and Neuman [2001] have shown that the same holds true for cross-hole tests and therefore adopt the simpler  $p$ -based representation, as we do here.

The full solution is given by

$$p_d(t_d) = \text{erfc}(w). \quad (1)$$

where  $w = 1/\sqrt{4t_d}$ . Here  $p_d$  is dimensionless pressure and is equal to  $4\pi krp/(q\mu)$  while  $k$  is permeability,  $r$  is the distance between the centroids of the injection and monitoring intervals,  $p$  is the change in pressure in the monitoring interval,  $q$  is the flow rate, and  $\mu$  is dynamic viscosity. The dimensionless time is defined as  $t_d = ktp_{\text{ave}}/(\phi\mu r^2)$ , where  $p_{\text{ave}}$  is average pressure and  $\phi$  is porosity. Using Abramowitz and Stegun (Eqn. 7.1.5),

$$p_d(t_d) = 1 - \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n w^{2n+1}}{n!(2n+1)}. \quad (2)$$

For small  $w$  (or large  $t_d$ ),

$$p_d(t_d) \approx 1 - \frac{2}{\sqrt{\pi}} w = 1 - \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{t_d}}. \quad (3)$$

Recalling the definitions of the dimensionless quantities  $p_d$  and  $t_d$ , we can write (1) as

$$p_d(t_d) = 1 - \frac{1}{\sqrt{\pi}} \sqrt{\frac{\phi\mu r^2}{ktp_{\text{ave}}}}. \quad (4)$$

For the change in pressure inside the monitoring interval, we can write (4) as

$$p = \left( \frac{q\mu}{4\pi kr} \right) - \left( \frac{q\mu}{4\sqrt{\pi}\pi k} \sqrt{\frac{\phi\mu}{kp_{\text{ave}}}} \right) t^{-1/2}. \quad (5)$$

We can see from (5) that  $p$  varies linearly with  $t^{-1/2}$ . The method thus requires plotting the change in pressure,  $p$ , at a given monitoring interval against values of the reciprocal of the square root of time ( $t^{-1/2}$ ). A straight line should develop for a portion of the data to which a straight line is fit. The intersection of this straight line with the time axis corresponding to  $t^{-1/2} = 0$  is denoted by  $p^*$ . The permeability of the formation is then determined from

$$k = \frac{q\mu}{4\pi rp^*}. \quad (6)$$

When  $p = 0$ , the straight line crosses the horizontal coordinate at some time  $t^*$ , which allows for the determination of  $\phi$ , using the formula

$$\phi = \frac{\pi k p_{\text{ave}} t^*}{\mu r^2} = \frac{q p_{\text{ave}} t^*}{4r^3 p^*}. \quad (7)$$

### 3 Application to three-dimensional pressure interference tests

We apply our technique to a set of three-dimensional pressure interference tests conducted at the Apache Leap Research Site (ALRS).

#### 3.1 Site and test description

The site was located near Superior, Arizona, at an elevation of 1,200 m above sea level. The test site included 22 vertical and inclined (at  $45^\circ$ ) boreholes that have been completed to a maximum depth of 30 m within a geologically distinct unit of partially welded unsaturated tuff. The upper 1.8 m of each borehole was cased. Core samples were taken from 9 of the 22 boreholes and a variety of tests were performed by Rasmussen et al. [1990] to determine the interstitial properties of the tuff matrix. Single-hole pneumatic and hydraulic injection tests were initially conducted by Rasmussen et al. [1990] with an injection interval length of 3 m to determine estimates of permeabilities of the fractured tuff. Guzman et al. [1996] then conducted over 270 single-hole pneumatic injection tests in 6 of the 22 boreholes with various injection interval lengths. Additional details to these tests and the site are provided in Rasmussen et al. [1990], Guzman et al. [1996], and Illman et al. [1998].

Core and single-hole pneumatic injection tests provide information only about a small volume of rock in the close vicinity of the injection interval. Fractured rock properties measured on such small scales tend to vary rapidly and erratically in space so as to render the rock strongly and randomly heterogeneous. To determine the properties of the rock on larger scales, Illman et al. [1998; see also Illman, 1999] conducted numerous cross-hole pneumatic injection tests between 16 boreholes (one of which included all 22 boreholes), 11 of which have been previously subjected to single-hole testing. The tests consisted of injecting air into an isolated interval within one borehole while monitoring pressure responses in isolated intervals within this and all other boreholes. The purpose of these tests was to determine the bulk pneumatic properties of larger rock volumes between boreholes at the site, and the degree to which fractures are pneumatically interconnected.

The tests were performed using modular straddle packer systems that were easily adapted to various test configurations and allowed rapid replacement of failed components, modification of the number of packers, and adjustment of distances between them in both the injection and monitoring boreholes. The main injection string consisted of three packers, one near the soil surface to isolate the borehole from the atmosphere, and two to enclose the injection interval. The air-filled volume of the injection interval was made relatively small so as to minimize borehole storage effects. Intervals with a single packer near the soil surface (of which we had six) are identified below by borehole designation; for example V1, X1 and W1. Where a modular system separates a borehole into three isolated intervals, we append to the borehole designation a suffix U, M or B to identify the upper, middle or bottom interval, respectively; for example V3U, V3M and V3B. Where a modular system separates a borehole into four isolated intervals, we append to the borehole designation a suffix U, M, L or B to identify the upper, middle, lower or bottom interval, respectively; for example Z2U, Z2M, Z2L, and Z2B.

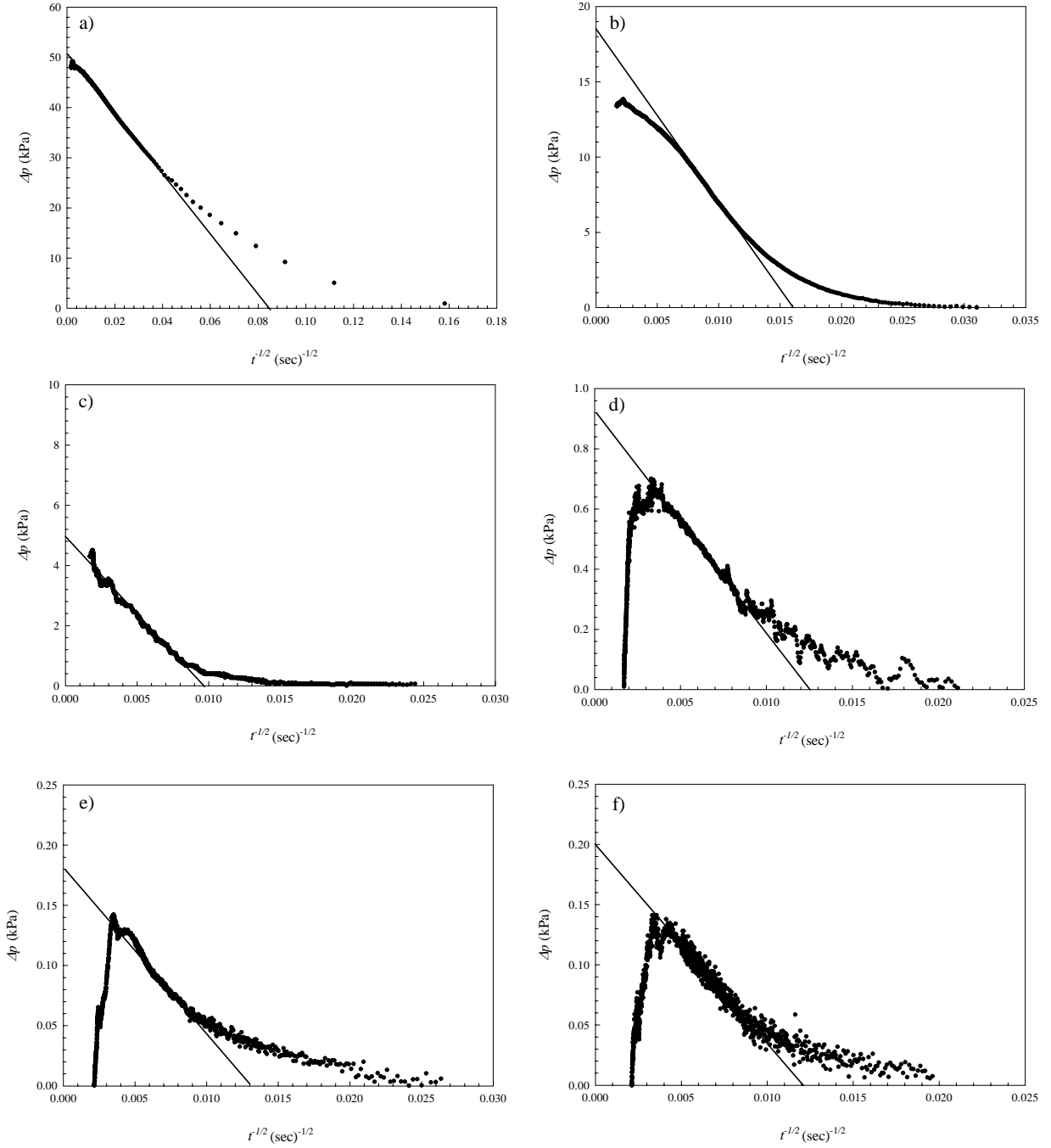


Figure 1: Results of asymptotic analysis of pressure transients in monitoring intervals a) W1, b) V2M, c) V1, d) W3M, e) W2AU, f) W2AM during cross-hole pneumatic injection test PP4.

A typical cross-hole test consisted of packer inflation, a period of pressure recovery, air injection and another period of pressure recovery. Our system allowed rapid release of packer inflation pressure when the corresponding recovery was slow, but this feature was never activated even though recovery had sometimes taken several hours. Once packer inflation pressure had dissipated in all (monitoring and injection) intervals, air injection at a constant flow rate began. It generally continued for several days until pressure in most monitoring intervals appeared to have stabilized. In some tests, injection pressure was allowed to dissipate until ambient conditions have been recovered. In other tests, air injection continued at incremental flow rates, each lasting until the corresponding pressure had stabilized, before the system was allowed to recover.

Three types of cross-hole tests were conducted at the ALRS in 3 phases. Phase 1 included line-injection/line-monitoring (LL) tests in which injection and monitoring took place along the entire length of a borehole that had been isolated from the atmosphere by means of shallow packers. Phase 2 consisted of point-injection/line-monitoring (PL) tests in which air was injected into a 2-m section in one borehole while pressure was recorded along the entire length of each monitoring borehole. During Phase 3, we conducted point-injection/point-monitoring (PP) tests in which both the injection and the monitoring intervals were short enough to be regarded, for purposes of type-curve analysis [Illman and Neuman, 2001], as points. A total of 44 cross-hole pneumatic interference tests of various types (constant injection rate, multiple step injection rates, instantaneous injection) have been conducted using various configurations of injection and monitoring intervals (LL, PL and PP).

## 3.2 Results

Recently, we have used this asymptotic approach to analyze data from various cross-hole pneumatic injection tests in unsaturated fractured tuff. The asymptotic analysis was conducted on tests deemed successful in that 1) they did not suffer from significant equipment failure and 2) their flow conditions were relatively well controlled and stable. We analyze selected data from 4 such tests (PP4-PP7) which were previously subjected to transient analyses by Illman and Neuman [2001] (PP4), and Vesselinov et al. [2001a-b] (PP4 – PP7) and steady-state analysis by Illman and Neuman [2003] (PP4-PP7). We apply the approach to pressure data in which both the injection and monitoring intervals are short enough to be regarded, for purposes of analysis, as points. Data from a large number of intervals were not amenable to our asymptotic analysis because the approximation of the point source solution applies only to data for which the point and monitoring intervals can be treated as points.

Figure 1 shows the results from analyzing 6 records of monitoring interval data from cross-hole test PP4. Details to the test are given in Illman and Neuman [2001]. It reveals that after an early time behavior that may be dominated by the effects of borehole storage, skin, and heterogeneity, a straight line develops. A visual examination of all pressure records reveals that all of them attain this straight line behavior at sufficiently large time and should therefore be amenable to our asymptotic analysis. We see that the latter part of test PP4 is affected by barometric pressure effects causing the pressure to decline, more so in the case when the signal-to-noise ratio is small. Figures 1a-c show that the signal-to-noise ratio is relatively large making the definition of the straight line portion of the pressure transients relatively easy. The data plotted in Figures 1d-f, on the other hand have a low signal-to-noise

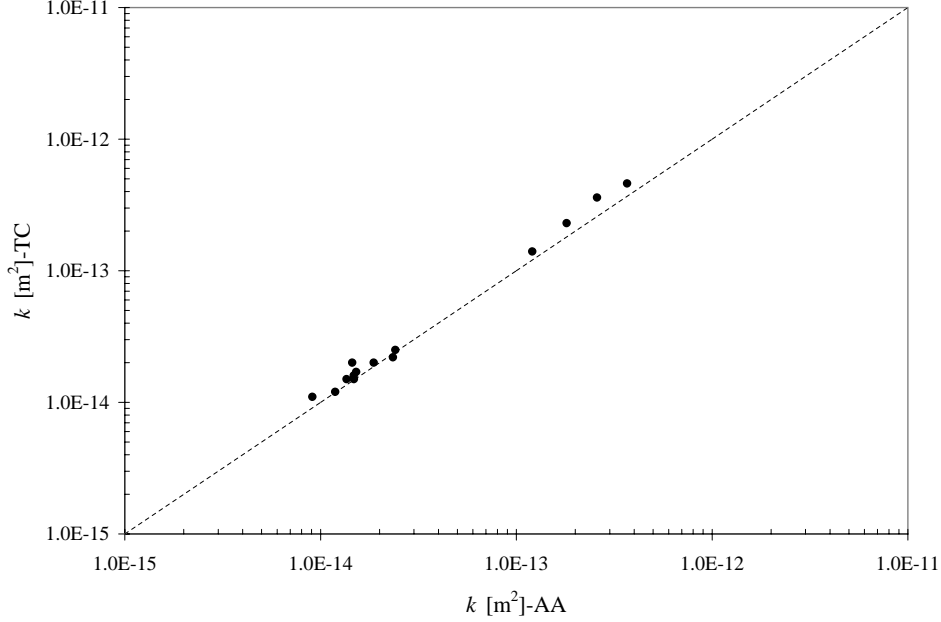


Figure 2: Comparison of permeabilities determined from the asymptotic analysis (AA) and type-curve analysis (TC) (Illman and Neuman, 2001) from test PP4 when  $\beta_1 > 5$  (point source case).

ratio making the definition of the straight line more difficult.

Log10-transformed permeability values from the asymptotic analysis range from -14.43 ( $3.73 \times 10^{-15} \text{ m}^2$ ) to -12.43 ( $3.68 \times 10^{-13} \text{ m}^2$ ) with a mean of -13.70 ( $1.98 \times 10^{-14} \text{ m}^2$ ), variance of 0.18, and coefficient of variation equal to -0.035. Likewise, the log10-transformed porosity values range from -2.59 ( $2.59 \times 10^{-3}$ ) to -1.00 ( $9.96 \times 10^{-2}$ ) with a mean of -1.77 ( $1.68 \times 10^{-2}$ ), variance of 0.19, and coefficient of variation equal to -0.248.

Our analysis of pressure transient data assumes that the rock is pneumatically uniform and isotropic on the scale of the cross-hole test. However, data from different monitoring intervals are seen to yield different values of pneumatic parameters, thereby providing information about their spatial and directional dependence. The values of permeabilities and porosities can be viewed as bulk directional properties of the rock associated with given injection and monitoring intervals.

## 4 Discussion

### 4.1 Comparison to results from type curve analysis

Illman and Neuman [2001] found it possible to interpret a single cross-hole pneumatic injection test labeled PP4 by means of analytically derived type-curves, based on a linearized version of the nonlinear partial differential equation that governs single-phase airflow in a

uniform, isotropic porous continuum while treating water as if it was immobile. Their type-curves represent a modification and extension of type-curves developed earlier by Hsieh and Neuman [1985] for the interpretation of hydraulic cross-hole tests in anisotropic rocks. The extension entails accounting for the combined effects of compressible air storage and skin in monitoring intervals, which they found to be of considerable importance during their cross-hole test (elsewhere they showed [Illman and Neuman, 2000] that air compressibility is the dominant factor affecting single-hole pneumatic injection tests at the ALRS, the corresponding skin effect being virtually zero). It was likewise important to include in their analysis type-curves of pressure derivative versus the logarithm of time, which accentuate phenomena such as the effect of barometric pressure, and aid in constraining the estimation of pneumatic parameters. A further improvement in their estimation of pneumatic rock properties was achieved by developing type-curves that allowed them to analyze pressure buildup and recovery data simultaneously.

Of the 44 cross-hole tests conducted at ALRS, Illman and Neuman [2001] were able to analyze only one of the cross-hole tests labeled, PP4. Tests during which injection took place into low to moderate permeability intervals proved to be more difficult to analyze by means of transient methods than those during which injection took place into high permeability intervals. This is so because the latter tests have generated distinct pressure signals that were relatively unaffected by background noise (due in large part to atmospheric pressure fluctuations) whereas the former tests have generated relatively weak pressure signals that were more difficult to separate from noise.

Permeability estimates obtained from the asymptotic analyses are compared to available type-curve estimates of permeabilities by Illman and Neuman [2001]. Figure 2 shows this comparison revealing that the comparison is excellent. However, the comparison of porosity estimates obtained (Figure 3) shows that some of the values compare well but there are several type-curve estimates that are heavily biased toward lower porosity values. The discrepancy comes from data collected in monitoring intervals located in boreholes Y3, Z2 and Z3. Illman and Neuman [2001] interpreted these data (for example, their Figure 10j) to have a very high observation wellbore storage causing the match to be shifted to the right to fit the early data with shallower slope. This we believe caused the porosity to be artificially smaller. The asymptotic analysis, on the other hand, relies on the intermediate data and these porosity estimates are more consistent with those found through the numerical inverse interpretation (discussed in section 4.3). There are other factors that can cause a shallower slope including the presence of high permeability features that connect the injection and monitoring intervals, but the type curve model of Illman and Neuman [2001] did not include such high permeability features.

Another factor that can cause nonunique estimates of porosity is the lack of match between the type-curve and early time data. Test PP5 was analyzed by means of type-curve methods described in Illman and Neuman [2001] but the majority of the early data failed to match the type-curves. One such example is shown in Figure 4, which demonstrates the pressure transients arrive later than the theoretical curves implying that there is a low permeability region between the injection and monitoring intervals. This causes the horizontal match to be nonunique even with the use of pressure derivatives and recovery techniques described in Illman and Neuman [2001] making the porosity determined from such matches highly unreliable. The asymptotic straight line analysis takes out this uncertainty in porosity



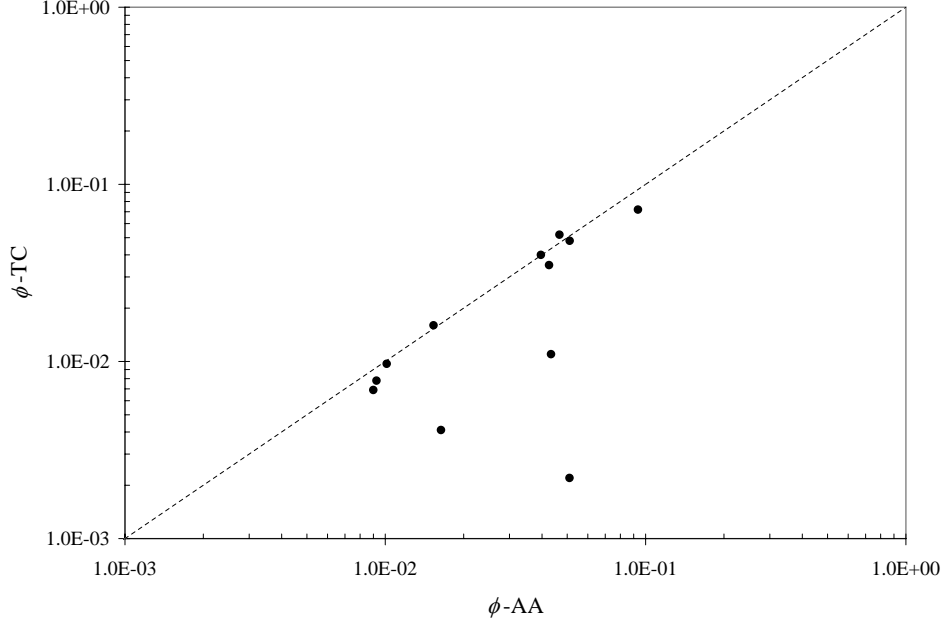


Figure 3: Comparison of porosities determined from the asymptotic analysis (AA) to those obtained by type-curve analysis (TC) Illman and Neuman (2001) from test PP4 when  $\beta_1 > 5$  (point source case).

estimates by focusing on the fit to the straight line portion of the data.

## 4.2 Comparison with results from steady-state analysis

The inability to analyze many cross-hole tests by means of type-curves led Illman and Neuman [2003] to use a steady-state formula developed by Hsieh and Neuman [1985] for hydraulic cross-hole tests in saturated rocks. Steady state analyses are much easier to conduct than transient type-curve [Illman and Neuman, 2000; Illman and Neuman, 2001] and numerical inverse [Vesselinov et al., 2001a-b] analyses, which have therefore been limited to relatively few single- and cross-hole tests. They found that their steady state approach to work well for pressure records whose signal-to-noise ratio is too low to allow meaningful transient analysis. They were therefore able to augment in a significant way the database previously established for the ALRS by other means. Though the steady state method does not yield estimates of porosity, it does yield reliable estimates of permeability between an injection and a monitoring interval. The results were analyzed statistically and they discussed their implications vis-a-vis the pneumatic properties of unsaturated fractured tuff at the ALRS. Their results strengthened the evidence for a previously surmised permeability scale effect at the site.

These results are compared against permeability estimates from the steady state analysis by Illman and Neuman [2003]. Figure 5 shows this comparison revealing that the comparison is quite good with a slight bias toward the steady state estimates of permeability. This may

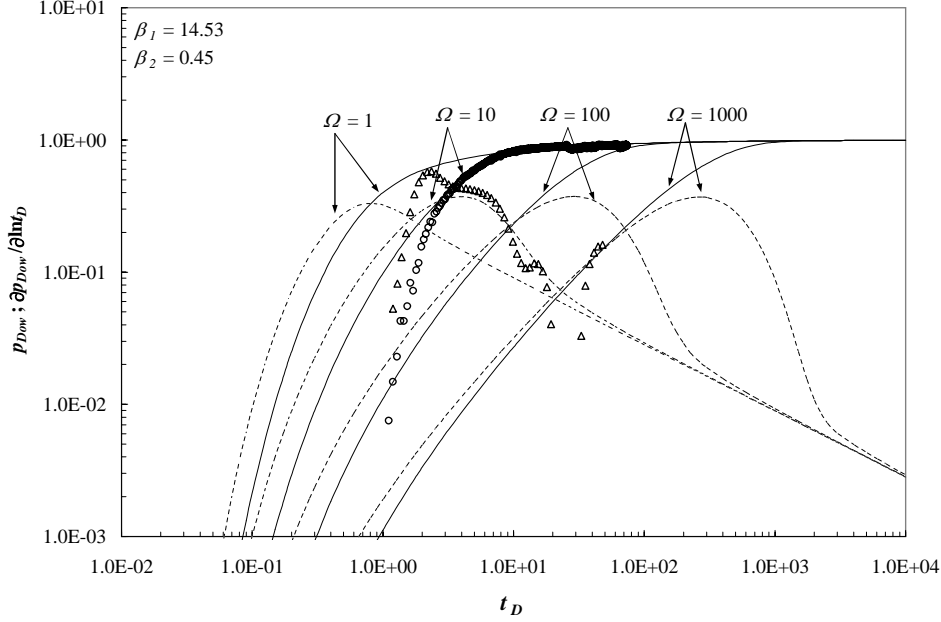


Figure 4: Pressure (open circles) and its derivative (open triangles) from monitoring interval V3M during cross-hole test PP5 matched against cross-hole type-curves of varying geometrical parameters ( $\beta_1, \beta_2$ ), and dimensionless well response time ( $\Omega$ ). Solid curve represents dimensionless pressure ( $p_{Dow}$ ), while dashed curve represents dimensionless pressure derivative ( $\partial p_{Dow} / \partial \ln t_d$ ).

be due to the fact that the steady-state estimates reflect a larger volume of the rock as the estimates are based on late data. Such a time dependence of permeability was observed by Schulze-Makuch et al [1998] through their analysis of pumping test data in fractured carbonates.

### 4.3 Comparison with results from numerical inverse analysis

We also compared our results to the available results from a three-dimensional numerical inverse interpretation of the same data [Vesselinov et al., 2001a-b]. The model simulates air-flow on a three-dimensional grid of structured and unstructured tetrahedral elements, which represents quite accurately the geometry of vertical and inclined boreholes at the ALRS. Boreholes are treated in the model as high-permeability and high-porosity cylinders of finite length and radius. The model treats permeabilities and porosities either as uniform throughout the rock volume or as random fractal fields. In the first case, the estimated parameters represent equivalent values over rock volumes having length-scales ranging from meters to tens of meters, represented nominally by radius vectors extending from the injection interval to the various monitoring intervals. In the second case, they describe the spatial variation of local pneumatic properties throughout the tested rock volume. In their model, this spatial

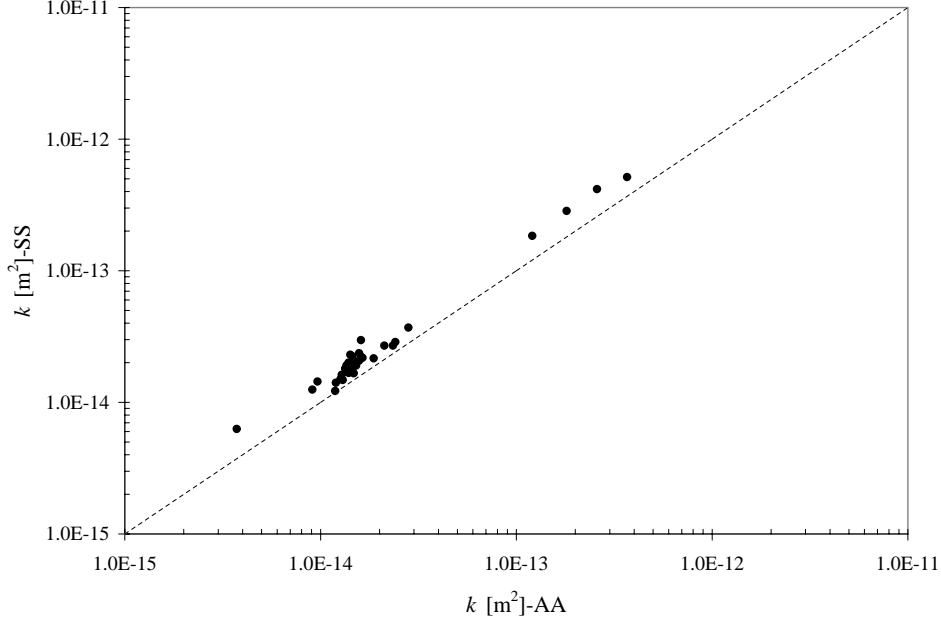


Figure 5: Comparison of permeabilities determined from the asymptotic analysis (AA) to those obtained by steady-state analysis (SS) (Illman and Neuman, 2003) of data from PP4-PP7 with  $\beta_1 > 5$ .

variability was characterized by a power variogram and was estimated geostatistically by Kriging, on the basis of discrete pilot points. Such estimation entailed the simultaneous inversion of pressure records from multiple observation intervals and cross-hole tests. It thus amounts to relatively high-resolution pneumatic tomography, or stochastic imaging, of the rock.

Vesselinov et al. [2001a,b] analyzed the data first one pressure record at a time making it analogous to the analytical interpretive techniques described here. They noted that each such numerical inversion required  $\sim 80$  forward simulations and it took  $\sim 4$  hours on the University of Arizona SGI Origin multiprocessor supercomputer. To interpret the cross-hole tests with the inverse model, [Vesselinov et al. 2001a,b] filtered the available pressure records so as to focus on signals that appear to be due primarily to air injection and to reduce the large set of recorded pressures done to a manageable number without the significant loss of information. They did so by ignoring those portions of a pressure record that they deemed strongly influenced by barometric pressure fluctuations or other extraneous phenomena and by representing the remaining portions via a relatively small number of "match points." The match points are distributed more or less evenly along the log-transformed time axis so as to capture with equal fidelity both rapid pressure transients at early time and more gradual pressure variations at later time. Matching was done with equal weighting using the match points with the numerical inverse interpretation.

The comparison of the permeabilities obtained from the inverse model treating the rock

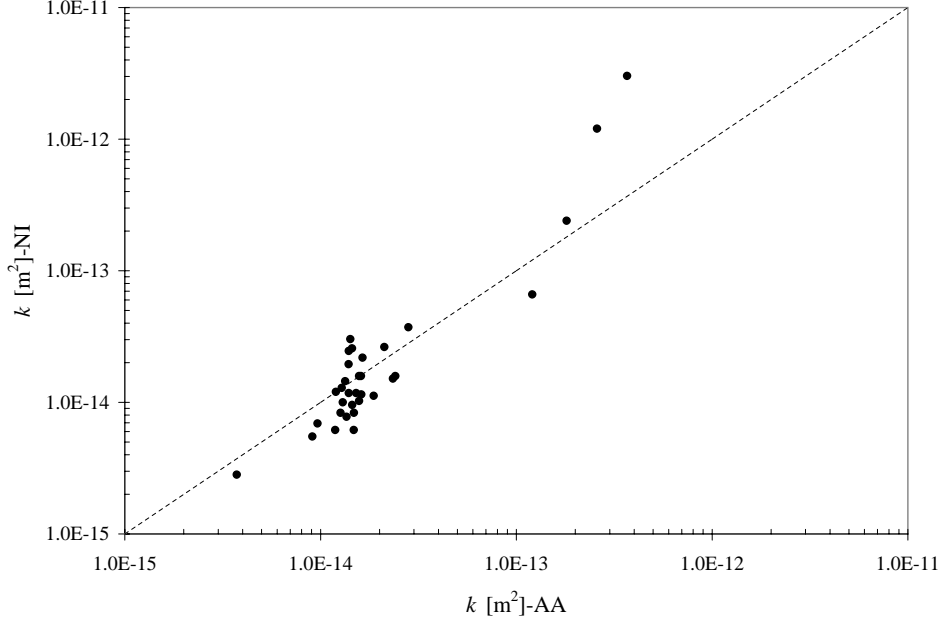


Figure 6: Comparison of permeabilities determined from the asymptotic analysis (AA) of data from PP4-PP7 with  $\beta_1 > 5$  to those obtained by the numerical inverse method of Vesselinov et al. (2001b) treating the rock as a homogeneous medium.

to be uniform and our results (Figure 6) shows that this comparison is good although the scatter is greater than in Figure 2. On the other hand, the comparison of porosity estimates (Figure 7) obtained shows a much larger scatter reflecting the fact that the porosity estimates are more uncertain. This is also reflected in the higher confidence intervals associated with the porosity estimates in comparison to the permeability estimates by means of the numerical inverse model [Vesselinov et al., 2001a-b].

#### 4.4 An alternative definition of pressure transient arrival time

The asymptotic analysis should be in principle readily incorporated into a numerical inverse model such as those by Yeh and Liu [2000], Vesselinov et al. [2001a,b] and Brauchler et al. [2003]. In particular, Brauchler et al. [2003] developed a numerical inverse model based on the arrival of pressure transients. They defined the travel time of the pressure peak when the pressure reaches a certain percentage of the maximum pressure (1, 2, 5, 10, 20, 30, and 40%). They conducted pneumatic tomography based on the arrival times of pressure transients which yielded a distribution of diffusivity for a laboratory sandstone sample. Their expression, originally derived by Vasco et al. [2000], is a line integral relating the arrival time of a "hydraulic signal" to the inverse of diffusivity. The line integral relates the square root of the drawdown peak arrival time of a transient pressure curve obtained for a Dirac source at the origin directly to the square root of the reciprocal value of the diffusivity.

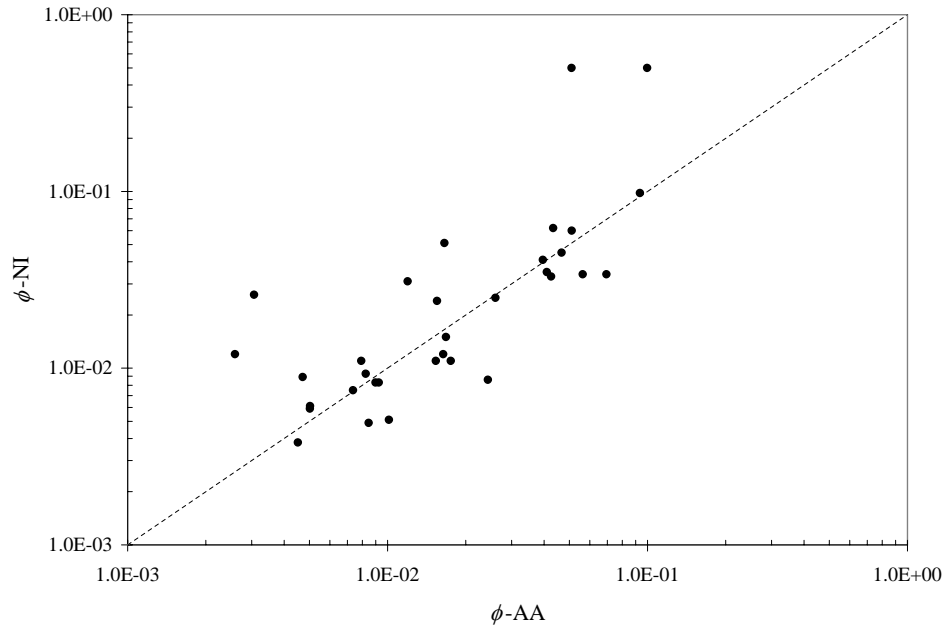


Figure 7: Comparison of porosities determined from the asymptotic analysis (AA) of data from PP4-PP7 with  $\beta_1 > 5$  to those obtained by numerical inverse method of Vesselinov et al. (2001b) treating the rock as a homogeneous medium.

Various researchers have defined the arrival times of pressure transients [Vasco et al. 2000; Brauchler et al., 2003]. These approaches are useful but the first arrival of pressure transient is difficult to define in many cases especially when the signal to noise ratio is small. The inaccurate definition of arrival times can translate into large errors in parameters that are estimated using the conventional arrival time approach. However, the arrival of the pressure transient during the intermediate to late period when a straight line develops is much more definitive. This is illustrated through records of pressure transients and changes in barometric pressure during cross-hole test PP5 from monitoring intervals W2AM (Figure 8a) and W1 (Figure 8b). In Figure 8a, the signal-to-noise ratio is relatively large making the definition of the arrival time of the pressure transient easier. In Figure 8b, the definition of the arrival time becomes more problematic because of the much smaller signal-to noise ratio. It is seen on Figure 8b that the pressure transient arrives at a very early time ( $\sim 100$  sec) but does not show a noticeable increase until about ( $\sim 10,000$  sec) after the test begins. Therefore, the analysis based on the first arrival of the pressure transients can cause large errors in parameter estimates. However, in both cases, the straight line portion of the pressure transient can be defined with good accuracy. Therefore, we propose that the arrival time of the pressure transient as the time at which the straight line intersects the abscissa.

## 5 Conclusions

This study leads to the following major conclusions:

1. Traditional methods of well test analysis rely on steady-state or transient methods. For the steady-state method, the pressure transient data collected during a pressure interference test must reach a steady-state for the method to be applicable. Likewise, for type-curve and numerical inverse approaches, the time-drawdown data must fit the model developed for the situation under consideration for the parameter estimates to be meaningful. In many cases these requirements are difficult to meet under field conditions due to external forcings and heterogeneities of the rock properties. Here, we develop a new approach to estimate permeability and porosity from well tests using the asymptotic analysis of pressure transients during three-dimensional pressure interference tests. The method is based on an asymptotic approximation of the point source solution which results in pressure varying linearly with  $t^{-0.5}$ . It merely requires plotting the data on a pressure versus the reciprocal of the square root of time which causes the pressure transient to develop a straight line regime. Major advantages of the approach include its simplicity and the lack of need of type-curves or numerical inverse models to obtain estimates of permeabilities and porosities.

2. We apply the technique to previously conducted cross-hole pneumatic injection tests by Illman et al., [1998; see also Illman, 1999] at the Apache Leap Research Site near Superior, Arizona, USA and compare these results to previously obtained estimates of permeabilities and porosities from type-curve [Illman and Neuman, 2001] and numerical inverse [Vesselinov et al. 2001a,b] analyses and permeabilities from steady-state [Illman and Neuman, 2003] analysis. The approach was applied to test data from monitoring intervals that could be analyzed using a point source solution. The comparisons reveal that the newly developed

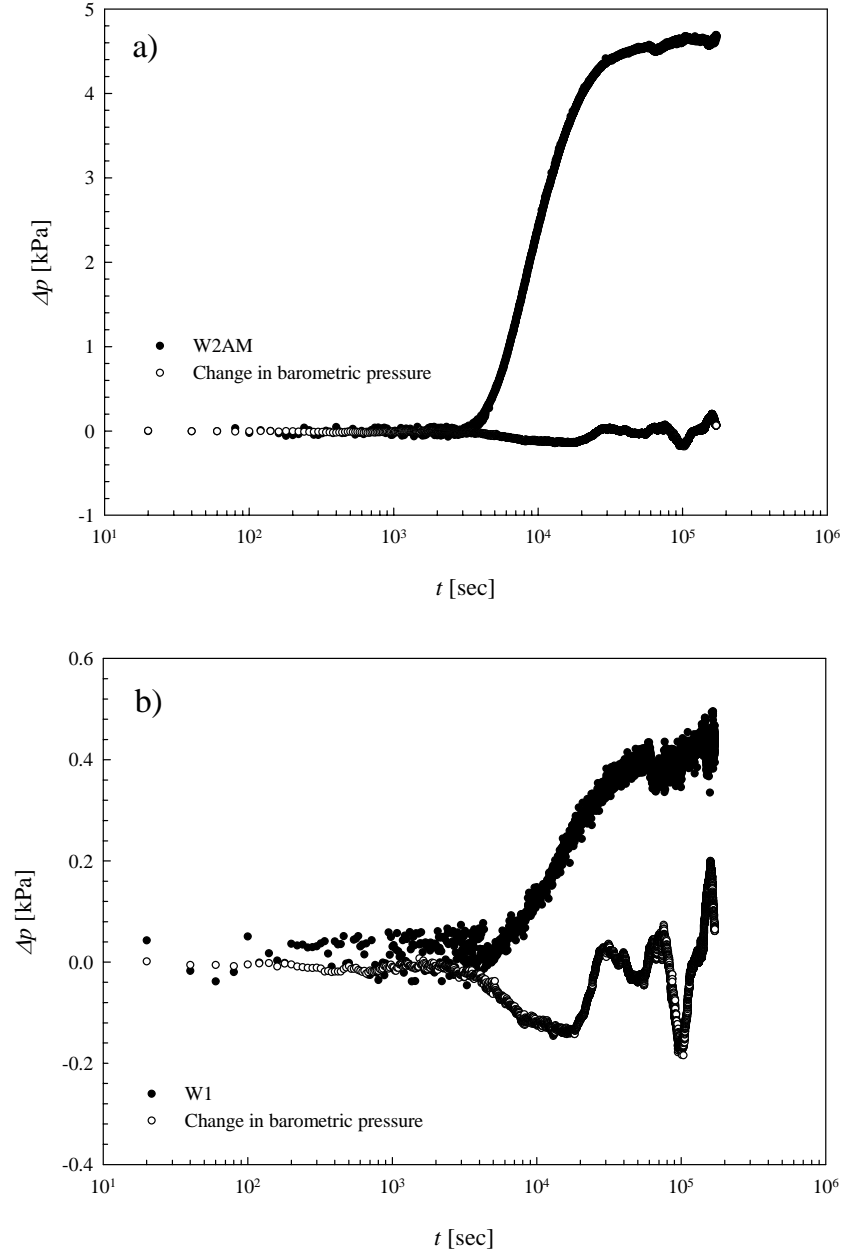


Figure 8: Records of pressure transients and changes in barometric pressure during cross-hole test PP5 from monitoring intervals W2AM (Figure 8a) and W1 (Figure 8b). In Figure 8a, the signal-to-noise ratio is relatively large making the definition of the arrival time easier. In Figure 8b, the definition of the arrival time becomes more problematic because of the much smaller signal-to noise ratio.

approach yields reliable estimates of permeabilities and porosities from three-dimensional pressure interference tests.

3. The asymptotic analyses are much easier to conduct than transient type-curve [Illman and Neuman, 2000; Illman and Neuman, 2001] and numerical inverse [Vesselinov and Neuman, 2001; Vesselinov et al., 2001a-b] analyses, which have therefore been limited to relatively few single-hole and cross-hole tests. We found our asymptotic approach to work well for pressure records whose signal-to-noise ratio is too low to allow meaningful transient analysis. This also includes cases when pressure transients are heavily affected by borehole storage, external forcings, and heterogeneities that cause the data to depart from analytically derived type-curve models. We were therefore able to augment in a significant way the database previously established for the ALRS by other means. In addition to estimates of permeability, the asymptotic approach yields reliable estimates of porosity between an injection and a monitoring interval, which cannot be obtained from the steady-state analysis of the same data.

4. Comparison of permeabilities from the asymptotic analysis to the type-curve analyses [Illman and Neuman, 2001] is excellent. However, the agreement between the asymptotic and type-curve estimates of porosities are not very good. This is because of the large uncertainty in porosity estimates from type-curve analysis resulting from borehole storage and subsurface heterogeneity that causes the early data to not match the type-curves making the horizontal match arbitrary. This is so despite the fact that pressure derivative analysis and recovery analyses were employed to conduct the type-curve analysis. The asymptotic analysis requires a fitting of a straight line to determine those values (quickly) and much more definitively than type curve analysis which requires considerable experience by the hydrogeologist.

5. Comparison of permeabilities between the asymptotic analysis and numerical inverse approach of Vesselinov et al. [2001a,b] was good. However there is some scatter in the data. Comparisons of the porosities showed increasing scatter suggesting the higher uncertainty in the parameter.

6. Comparison of permeabilities obtained from the asymptotic to steady state analysis is good although the permeabilities are slightly biased toward higher permeability values for the steady-state approach. This may be due to the fact that the steady state portion of the pressure transient has sampled a larger portion of the rock giving rise to a larger effective permeability. We emphasize that the steady-state analysis does not yield estimates of porosity but the asymptotic analysis does.

7. The approach we present here should be readily incorporated into a numerical inverse model to obtain estimates of permeabilities and porosities. As the pressure transients from early time are difficult to analyze because of the low signal to noise ratio, the arrival time analysis may be more readily conducted with the arrival of the straight line portion of the pressure transients.

8. Our analysis of pressure transient data assumes that the rock is pneumatically uniform and isotropic on the scale of the cross-hole test. Results from individual monitoring intervals



provided information about pneumatic connections between these and the injection interval, corresponding directional permeabilities, and porosities. Each pressure record yielded an equivalent directional permeability and porosity for fractures that connect the corresponding monitoring and injection intervals. Both quantities were found to vary considerably from one pressure monitoring record to another. Thus, even though our asymptotic analysis treats the rock as if it was pneumatically uniform and isotropic, it ultimately yields information about the spatial and directional dependence of pneumatic connectivity, permeability and porosity of fractures across the site on scales relevant to the cross-hole test.

## 6 Acknowledgements

The first author was supported in part by the 2003 Old Gold Fellowship from the University of Iowa, as well as by funding from the National Science Foundation (NSF) and the Strategic Environmental Research & Development Program (SERDP). The research by the second author was performed under the auspices of the U.S. Department of Energy, under contract W-7405-ENG-36; and was supported in part by the LDRD Program at Los Alamos National Laboratory.

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